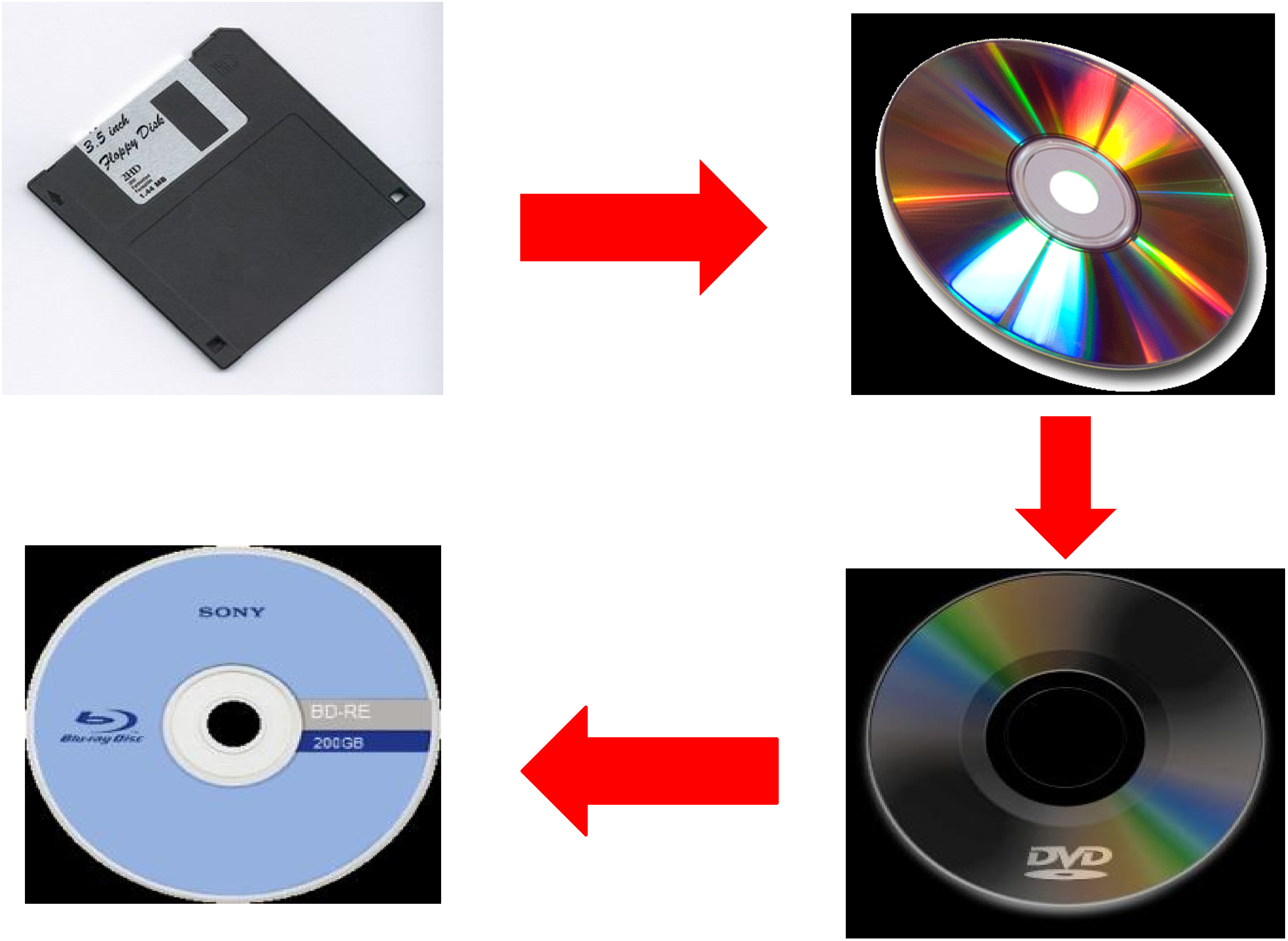
Replacement Problems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Introduction:

The efficiency of all industrial and military equipments deteriorates with time. Sometimes the equipment fails completely and effects the whole system. The maintenance costs (running costs) of an equipment also go on increasing with time. Thus it becomes more economical to replace the old equipment with a new one. Hence there is a need to formulae a most economical replacement policy which is in the best interest of the system.

There are following types of Replacement problems under discussion:

1. Replacement of major or capital item (equipment) that deteriorates with time; e.g., Machines, Trucks etc.

2. Replacement of an item (equipment) in anticipation of complete failure, the probability of which increases with time (or age).

3. Problems in morality and staffing.

4. Replacement of an equipment (or item) may be necessary due to new researches, otherwise the system may become out of date.

Replacement of major or capital item (equipment) that deteriorates with time; e.g., Machines, Trucks etc.

For machine the maintenance cost always increase with time and stage comes when the maintenance cost becomes so large that it is economical to replace the machine with a new one. Thus the problem of replacement in this case is to find the best time (age of machine) at which the old machine should be replaced by the new one.

To Find The Best Replacement Age (Time) of a Machine When

(1) Its maintenance cost is given by a function increasing with time.

(2) Its scrap value is constant and,

(3) The Money value is not considered.

Let C = Cost of Machine

S = Scrap value of the machine at time t.

and, $C_m(t)$ = Maintenance cost of machine at time t, [$C_m(t)$ increases with t].

Case 1 : When time 't' is a continuous variable.

Total maintenance cost of the machine in n years $= \int_0^n C_m(t)\,dt$

The total cost of machine in n years

= Cost of machine – Scrap value of machine + Total maintenance cost in n years.

$$= C - S + \int_0^n C_m(t)dt$$

The average cost of maintenance per year during n years is given by

$$A(n) = \frac{C - S + \int_0^n C_m(t)dt}{n}$$

$$A(n) = \frac{C - S}{n} + \frac{1}{n}\int_0^n C_m(t)dt \text{......................}(1)$$

Now for A (n) to be minimum,

$$\frac{d}{dn}\left(A(n)\right) = -\frac{C - S}{n^2} - \frac{1}{n^2}\int_0^n C_m(t)dt + \frac{1}{n}C_m(t)$$

$$\therefore C_m(t) = \frac{C - S + \int_0^n C_m(t)dt}{n} = A(n) \quad \text{[from (1)]}$$

i.e. Maintenance cost of the machine at time t

= Average cost of the machine per year.

Also $$\frac{d^2A(n)}{dn^2} = \frac{2(C-S)}{n^3} + \frac{2}{n^3}\int_0^n C_m(t)\,dt > 0$$

Hence, the average cost of the machine per year is minimized by replacing it when the average cost to date becomes equal to the current maintenance cost.

Case 2 : When time 't' is a discrete variable.

When time t takes the values 1, 2, 3,.....etc., then the total maintenance cost of the machine in n years

$$= \sum_{m=1}^{n} C_m(t)$$

Therefore, in this case the average cost of the machine per year during n years is given by

$$A(n) = \frac{C - S + \sum_{m=1}^{n} C_m(t)}{n} = \frac{C-S}{n} + \frac{1}{n}\sum_{m=1}^{n} C_m(t) \ldots\ldots\ldots(2)$$

But A(n) is minimum for that value of n for which

Δ (An -1) < 0 < Δ A (n)........................(3)

Δ A(n) = A(n + 1) – A(n)

$$= \left[\frac{C-S}{n+1} + \frac{1}{n+1}\sum_{m=1}^{n+1} C_m(t)\right] - \left[\frac{C-S}{n} + \frac{1}{n}\sum_{m=1}^{n} C_m(t)\right]$$

$$= (C - S)\left(\frac{1}{n+1} - \frac{1}{n}\right) + \left(\frac{1}{n+1}\right)\left[\sum_{m=1}^{n} C_m(t) + C_{m+1}(t)\right] - \frac{1}{n}\sum_{m=1}^{n} C_m(t)$$

$$= -\frac{(C - S)}{n(n+1)} + \left(\frac{1}{n+1} - \frac{1}{n}\right)\sum_{m=1}^{n} C_m(t) + \frac{1}{n+1} C_{m+1}(t)$$

$$= -\left[\frac{(C - S) + \sum_{m=1}^{n} C_m(t)}{n(n+1)}\right] + \frac{1}{n+1} C_{n+1}(t)$$

$$= -\frac{A(n)}{n+1} + \frac{1}{n+1} C_{n+1}(t)\ from...(2)$$

$$\Delta A(n-1) = -\frac{A(n-1)}{n} + \frac{1}{n} C_n(t)$$

From (2) A(n) is minimum for value of n for which
$\Delta A(n) > 0$ and $\Delta A(n-1) < 0$

i.e., $$-\frac{A(n)}{n+1}+\frac{1}{n+1}C_{n+1}(t)>0$$

and $$-\frac{A(n-1)}{n}+\frac{1}{n}C_{n}(t)<0$$

or $$C_{n+1}(t)>A(n)$$

and $$C_{n}(t)<A(n-1)$$

From (4), we conclude that

1. **Do not replace if the next years maintenance cost is less than the previous years average total cost.**
2. **Replace if the next years maintenance cost is greater than the previous years average total cost.**

A Machine is continued up to the time the average cost per year (including cost of the new machine, maintenance cost etc.) of the machine decrease and is replaced at the time when its cost begins to increase.

Case 1:

The cost of a bike is $ 3000. The salvage value (resale value) and the running cost are given as under. Find the most economical replacement age of the truck.

Table 1

Year	1	2	3	4	5	6	7
Running Cost	600	700	800	900	1000	1200	1500
Resale value	2000	1333	1000	750	500	300	300

Solution:

Table 2

Age of Replacement (in Years)	Total Running Cost ($)	Depreciation = Cost of bike – Resale Value ($)	Total Cost ($)	Average Cost per year ($)
1	600	1000	1600	1600.00
2	1300	1667	2967	1483.50
3	2100	2000	4100	1366.66
4	3000	2250	5250	1312.50
5*	4000	2500	6500	1300.00*
6	5200	2700	7900	1316.67

Replace the bike in 5th year.

Case 2:

Machines in a factory have increased cost as they continue in service due to increased running cost. The initial running cost is Rs 3,500 and resale price drops as time passes until it reaches a constant value of Rs 500. Determine the proper length of service before machines should be replaced. Cost data are given below.

Table 3

Year of Service	1	2	3	4	5
Running Cost	1800	2200	2700	3200	3700
Resale value	1900	1050	600	500	500

Solution:

Year of Service	*Running Cost*	*Cumulative Running Cost*	*Resale Price*	*Difference between Initial Cost and Resale Price*	*Average Investment cost per Year*	*Average Running Cost per Year*	*Average Cost Per Year*
1	2	3	4	5	6 = 5/1	7 = 3/1	8 = 6 + 7
1	1800	1800	1900	1600	1600	1800	3400
2	2200	4000	1050	2450	1225	2000	3225
3	2700	6700	600	2900	967	2233	3200
5	3700	13600	500	3000	600	2720	3320

Replace the machine in 4th year.

Case 3:

A. Machine A costs Rs 9000. Annual operating cost are Rs 200 for the 1st year, and then increase by Rs 2000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owing and operating the machine ? (Assume that the machine has no resale value when replaced, and that future cost are not discounted).

B. Machine B costs Rs 10,000. Annual operating costs are Rs 400 for the first year, and then increase by Rs 800 every year. You have now a machine of type A which is one year old. Should you replace it with B, and if so when ?

C. Suppose you are, just ready to replace machine A with another machine of the same type, when you hear that machine B will become available in a year. What you should do ?

Solution:

a. The maintenance cost of machine A per year are as follows:

Table 3

Year	1	2	3	4	5
Maintenance cost in Rs	200	2200	4200	6200	8200

To find the average cost per year of machine A, we prepare the following table:

Table 4

Replacement at the end of the year	Total Running Cost Rs	Depreciation Rs	Total Cost Rs	Average Cost Rs
1	200	9000	9200	9200
2	2400	9000	11400	5700
3*	**6600**	**9000**	**15600**	**5200**
4	12,800	9000	21,800	5450

It is clear from the table that the machine A should be replaced at the end of the 3rd year. The average yearly cost in this situation will be Rs 5200.

b. The maintenance cost of machine B per year are as follows:

Table 5

Year	1	2	3	4	5	6	7
Maintenance cost in Rs	400	1200	2000	2800	3600	4400	5200

To find the average cost per year of machine A, we prepare the following table:

Table 6

Replacement at the end of the year	Total Running Cost Rs	Depreciation Rs	Total Cost Rs	Average Cost Rs
1	400	10000	10400	10400.00
2	1600	10000	11600	5800.00
3	3600	10000	13600	4533.33
4	6400	10000	16400	4100.00
5*	**10000**	**10000**	**20000**	**4000.00**
6	14400	10000	24400	4066.66

It is clear from the above table that if machine B is replaced after 5 years then its average cost per year is Rs 4000.

Since the lowest average cost Rs 4000 for machine B is less than the lowest average cost Rs 5200 for machine A, the machine A should be replaced by machine B.

To find the time of replacement of machine A by machine B.

The machine A is replaced by machine B at the time (age) when its running cost of the next year exceeds the lowest average yearly cost Rs 4000 of machine B.

Total cost of machine A in the successive years are as follows:

Table 7

Year	1	2	3	4	5
Total cost in the year	9200	Rs 11400 – 9200 = Rs 2200	Rs 15600 – 11400 = Rs 4200	Rs 21800 – 15600 = Rs 6200	Rs 30000 – 21800 = Rs 8200

The running cost of third year of machine A is Rs 4200 which is more than the lowest average yearly cost Rs 4000 of machine B. Therefore, the machine A should be replaced by machine B when its age is 2 years. Since the machine A is one year old now, therefore it should be replaced after one year from now.

c. As shown in (b), the machine A should be replaced after one year from now and the machine B also available at that time. Therefore, the machine A should be replaced by machine B after one year from now.

Case 3:

For a machine, from the following data are available:

Table 8

Year	0	1	2	3	4	5	6
Cost of spares (Rs)	-	200	400	700	1000	1400	1600
Salary Maintenance staff (Rs)	-	1200	1200	1400	1600	2000	2600
Losses due to break down (Rs)	-	600	800	700	1000	1200	1600
Resale value	12000	6000	3000	1500	800	400	400

Determine the optimum period for replacement of the above machine.

Few Important Terms:

Money Value:

The time value of money is the value of money figuring in a given amount of interest earned over a given amount of time.

The value of money changes with time. The statement is explained by the following example.
If we borrow Rs 100 at interest of 10% per year then after one year after one year we have to return Rs 110. Thus Rs 110 after one year from now are equivalent to Rs 100 today. Therefore, Rs 1.00 after one year from now is equivalent to Rs 100/110 i.e., $(1.1)^{-1}$ today at the rate of 10% So, Rs 1.00, n year from now is equivalent to Rs $(1.1)^{-n}$ today at the rate of 10% thus we can say that the value of money changes with time.

Present Value or Present Worth:

We have shown that Rs 100, n years after from now is equivalent to Rs $(1.1)^{-n}$ today at the rate of 10%. In other words we say that $(1.1)^{-n}$ is the **present value (or present worth)** of one rupee spent after one year from now.

In general if the interest on Rs 1 is Rs '*i*' per year, then the present value of Rs 1 to be spent after n years from now is Rs $1/(1+i)^n$

Discount Rate or Depreciation Ratio:

The value of the money decreases with a constant ratio which is known as its **discount rate or depreciation ratio**.

If r is the ratio of interest per year on Rs 1.00. Then the present value of Rs 1.00 to be spent after one year from now is Rs $1/(1+r)$.

The ratio $v = \frac{1}{(1+r)}$ is the **discount rate** or **depreciation ratio** which is less than one.

To determine the best replacement age of items whose maintenance cost increase with time and the value of Money also changes with time:

$$\frac{1-v^{n-1}}{1-v}.C_n - P(n-1) < 0 < \frac{1-v^{n-1}}{1-v}.C_{n+1} - P(n)$$

Thus, the best replacement age of machine is n years for which the above inequality holds.

1. Do not replace if the operating cost of next period (year) is less than the weighted average of previous cost.
2. Replace if the operating cost of the next period (year) is greater than the weighted average of the previous cost.

Case: The cost pattern for two machines A and B when money value is not considered, is given as follows:

Table 9

Year		
	Machine A	Machine B
1 2 3	900 600 700	1400 100 700
Total	2200	2200

Find the cost for each machine, when money is worth 10% per year, and hence find which machine is less costly.

Solution :

The total expenditure for each machine in 3 years is equal to Rs 2200 when the money value is not taken in to consideration. Thus, the two machines are equally good if the money has no value over time.

When money value is 10% per year the discount rate

$$v = \frac{100}{100+10} = 0.9091$$

The present value of the maintenance cost in years 1, 2, 3 for two machines are shown in following table

Table 10

Year	Machine A (Rs)	Machine B (Rs)
1 2 3	900 600 × (0.9091) = 545.45 700 × $(0.9091)^2$ = 578.52	1400 100 × (0.9091) = 90.91 700 × $(0.9091)^2$ = 578.52
Total	Rs 2023.97	Rs 2069.43

The present value of the total expenditure for machine A in three years is less than that of machine B. hence machine A is less costly.

Case :

Let the value of money be assumed to be 10% per year and suppose that the machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machines are given as under.

Table 11

Year	1	2	3	4	5	6
Machine A :	1000	200	400	1000	200	400
Machine B :	1700	100	200	300	400	500

Determine which machine should be purchased.

Solution :

Since the value of money is 10% per year, the discount rate $v = \frac{100}{100+10} = \frac{10}{11}$

$\therefore$ The total discount cost (present worth) of costs of machine A for 3 years is

$$= 1000 + 200 \times \left(\frac{10}{11}\right) + 400 \times \left(\frac{10}{11}\right)^2 = Rs\ 1512 \quad app$$

and the total discount cost (present worth) of costs of machine B for 6 years is

$$= 1700 + 100 \times \left(\frac{10}{11}\right) + 200 \times \left(\frac{10}{11}\right)^2 + 300 \times \left(\frac{10}{11}\right)^3 + 400 \times \left(\frac{10}{11}\right)^4 + 500 \times \left(\frac{10}{11}\right)^5 = Rs\, 2765\, app$$

$\therefore$ The average yearly cost of Machine A = 1512/3 = Rs 504
And average yearly cost of machine B = 2765/6 = Rs 460.83

From here machine B looks less costly but its not true because the time periods considered are not the same

Now the total discount cost of costs of machine A for 6 years

= 1000 + 200 × (10/11) + 400 × $(10/11)^2$ + 1000 × $(10/11)^3$ + 200 × $(10/11)^4$ + 400 × $(10/11)^5$ = Rs 2647 app

The present value of the total expenditure in 6 years on machine A is less than that for machine B. Thus the machine A is less costly and hence machine A should be purchased.

A pipeline is due for repairs. It will cost ` 10,000 and lasts for three years. Alternatively, a new pipeline can be laid at a cost of ` 30,000 and lasts for 10 years . Assuming cost of capital to be 10% and ignoring salvage value, which alternative should be chosen.

Consider two types of pipelines for infinite replacement cycles of 10 years for the new pipeline, and 3 years for existing pipeline.

The **present worth factor** is given by; $v = \frac{100}{100+10} = 0.09091$

Let D_n denote the discounted value of all future costs associated with a policy of replacing the equipment after n years. Then if we designate the initial outlay by C,

$$D_n = C + v^n C + v^{2n} C + \ldots = C\left[1 + v^n + v^{2n} + \ldots\right] = C\left(1 - v^{-n}\right) = C / \left(1 - v^n\right)$$

$$D_n = \frac{C}{\left(1 - v^n\right)}$$

$$D_3 = \frac{10,000}{\left(1 - (0.9091)^3\right)} = Rs40021$$

and,

$$D_{10} = \frac{30,000}{\left(1 - (0.9091)^{10}\right)} = Rs48820$$

$$D_3 < D_{10}$$

The existing pipeline should be continued.

Case 4:

A manufacturer offered two machines A and B. A is priced at Rs 5000 and running cost are estimated at Rs 800 for each of the first 5 years, increasing by Rs 200 per year in the 6th and subsequent years. Machine B, which has the same capacity as A, costs Rs 2500 but with running cost of Rs 1200 per year for the first six years, increasing by Rs 200 per year there after. If money is worth 10% per year, which machine should be purchased?
(Assume that the machine will eventually be sold for scrap at negligible price.)

Solution:

If n is the optimum replacement age of the machine we have

$$\frac{1-v^{n-1}}{1-v}.C_n - P(n-1) < 0 < \frac{1-v^{n-1}}{1-v}.C_{n+1} - P(n)\text{......... ..1}$$

Where, $P(n) = A + C_1 + C_2v + C_3v^2 + \ldots\ldots + C_nv^{n-1}$

Here, v = 0.9091, 1 – v = 1/11.
For machine A, A = Rs 5000
To find the value of n, satisfying (1) for machine A, we compute this table.

Table 12

Year n	C_n	$v^{n-1}=(0.909)^{n-1}$	$(v^{n-1})\ C_n$	$(1-v^{n-1})\ C_n$	$\frac{(1-v^n)}{1-v} C_{n+1}$	P(n)	$\frac{(1-v^n)}{1-v} C_{n+1} - P(n)$
1	2	3	4 = 2 × 3	5 = 2 – 4	6	7	8 = 6 – 7
1	800	1.0000	800	0	800	5800	< 0
2	800	0.9091	727	73	1529	6527	< 0
3	800	0.8264	661	139	2189	7188	< 0
4	800	0.7513	601	199	2794	7789	< 0
5	800	0.6830	546	254	4169	8335	< 0
6	1000	0.6209	621	379	5753	8956	< 0
7	1200	0.5645	677	523	7502	9633	< 0
8	1400	0.5132	718	682	9394	10351	< 0
9*	1600	0.4665	746	854	11407	11097	> 0
10	1800	0.4241	763	1037	-	-	

Form the table, it is clear that equation (1) satisfied for n = 9.

∴ It would be best to replace machine A after 9 years. The fixed annual payment for machine A

$$x_1 = \frac{1-v}{1-v^9}.P(9)$$

$$= \frac{1-0.9091}{1-(0.9091)^9} \times 11097 = \frac{0.0909}{1-0.4241} \times 11097$$

$$= Rs\,1752$$

Again for machine B, A = Rs 2500

To find the value of n satisfying (1) for machine B, we compute thee following table

Table 13

Year n	C_n	$v^{n-1}=(0.909)^{n-1}$	$(v^{n-1})\ C_n$	$(1-v^{n-1})\ C_n$	$\frac{(1-v^n)}{1-v} C_{n+1}$	P(n)	$\frac{(1-v^n)}{1-v} C_{n+1} - P(n)$
1	2	3	4 = 2 × 3	5 = 2 – 4	6	7	8 = 6 – 7
1	1200	1.0000	1200	0	1200	3700	< 0
2	1200	0.9091	1091	109	2288	4791	< 0
3	1200	0.8264	992	208	3278	5783	< 0
4	1200	0.7513	902	298	4180	6685	< 0
5	1200	0.6830	820	380	5005	7505	< 0
6	1200	0.6209	745	455	6710	8250	< 0
7	1400	0.5645	790	610	8369	9040	< 0
8*	1600	0.5132	821	779	10560	9861	> 0
9	1800	0.4665	840	960	-	-	

Form the table, it is clear that equation (1) satisfied for n = 8.

∴ It would be best to replace machine B after 8 years. The fixed annual payment for machine A

$$x_2 = \frac{1 - v}{1 - v^8} . P(8)$$
$$= Rs\ 1680$$

Since $x_1 > x_2$
It would be better to purchase machine B instead of A.

Replacement of the machines in anticipation of complete failure the probability of which increases with time:

In general the probability of failure of an item increases with time. The complete failure of an item (machine) may result in complete break down of the system which puts the organization to a heavy loss. Thus there is a need of some replacement policy of such items to avoid the possibility of complete breakdown. Such items may be replaced even when they are in working order.
The following policies are followed:

1. Individual Replacement Policy:

According to this policy the failed item is immediately replaced by the new one, e.g. in house fused bulb is replaced by the new one.

1. Group Replacement Policy:

In general a system contains a large number of identical low – cost items such that the probability of their failure increases with time or age. In such cases there is a set up cost for replacement which is independent of the number replaced. Here in such systems it may be advantageous to replace all the items at some fixed interval. Such policy is called the group replacement policy. This policy is followed in the systems where the cost of an individual item is so small that the cost of keeping records of individual's age can not be justified.

To determine the interval of optimum replacement:

Group replacement should be made at the end of nth period if the cost of individual replacement for the n – th period is greater than the average cost per period by the end of n periods.

Group replacement should not be made at the end of nth period if the cost of individual replacement at he end of (n - 1)th period is not less than the average cost per period by the end of (n - 1) periods.

Problems in Morality:

The problems are the special cases of the problems in industry where the failure of any item (machine) can be treated as death and the replacement of any item (machine) can failure can be taken to a birth in the human populations. We consider the following morality problem:

Let a large population be subjected to a given morality curve for a very long period of time under the following assumptions.

(i) All death's are immediately replaced by births.

(ii) There are no other entries or exit.

Then we shall prove the following results

(i) The number of deaths per unit time becomes stable.

(ii) The age distribution ultimately becomes stable.

Staffing Problem:

The problems connected with recruitment and promotion of the staff in any system (organization)are known as the staffing problems. These problems are also treated as the replacement problem where the staff of the system is treated like a machine part. The problem of staff replacement arises due to resignation, retirements deaths of the staff members from time to time. Therefore to maintain a suitable strength of the staff in a system there is a need of some useful recruitment policy.

Morality Tables:

Morality tables for any item can be used to obtain the probability distribution of its life span.

Let N = The total number of items in the system in the beginning

And N (t) = Number of survivors at any time t.

Then the probability that any item will fail in the time interval (t – 1, t) is given by

$$\frac{N(t-1) - N(t)}{N} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

since N (t - 1) > N (t)

And the probability that any item that survived up to the age (t - 1) will die in the next year is given by

$$\frac{N(t-1) - N(t)}{N(t-1)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Case:

The following morality rates have been observed for a certain type of light bulb:

Week	1	2	3	4	5
Percent failing by the end of the week	10	25	50	80	100

There are 1000 bulbs in use and it costs Rs 1.00 to replace an individual bulb which has burnt out. If all bulbs were replaced simultaneously it would cost 25 paise per bulb. It is proposed to replace all bulbs at fixed intervals, whether or not they have burnt out and to continue replacing burnt out bulbs as they fail. At what intervals should all the bulbs be replaced?

Solution:

If P_1 is the probability of failure of a new bulb in i^{th} week, then

$$P_1 = \frac{10}{100} = 0.10$$

$$P_2 = \frac{25-10}{100} = 0.15$$

$$P_3 = \frac{50-25}{100} = 0.25$$

$$P_4 = \frac{80-50}{100} = 0.30$$

$$P_5 = \frac{100-80}{100} = 0.20$$

$$Here, P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

Since the sum of all probabilities is equal to 1.

$\therefore$ $P_6, P_7, \ldots\ldots$ Are all zero.

Therefore, a bulb can not survive for more than five weeks i.e., a bulb which has survived for four weeks is sure to fail in the fifth week.

Assuming that the burnt out bulbs in any week are replaced just at the end of that work. If N_i be the number of replacements at the end of i^{th} week, while all 1000 bulbs were new initially, then we have

N_0 = Number of bulbs in the beginning. = 1000

N_1 = Number of burnt out bulbs replaced at the end of 1^{st} week

$= N_0P_1 = 1000 \times 0.10$ = 100

N_2 = Number of burnt out bulbs replaced at the end of 2^{nd} week

$= N_0P_2 + N_1P_1 = 1000 \times 0.15 + 100 \times 0.10$ = 160

N_3 = Number of burnt out bulbs replaced at the end of 3^{rd} week

$= N_0P_3 + N_1P_2 + N_2P_1 = 1000 \times 0.25 + 100 \times 0.15 + 160 \times 0.10$ = 281

N_4 = Number of burnt out bulbs replaced at the end of 4^{th} week

$= N_0P_4 + N_1P_3 + N_2P_2 + N_3P_1$

$= 1000 \times 0.30 + 100 \times 0.25 + 160 \times 0.15 + 281 \times 0.10$ = 377

N_5 = Number of burnt out bulbs replaced at the end of 5^{th} week

$= N_0P_5 + N_1P_4 + N_2P_3 + N_3P_2 + N_4P_1$

$= 1000 \times 0.20 + 100 \times 0.30 + 160 \times 0.25 + 281 \times 0.15 + 377 \times 0.10$ = 350

N_6 = Number of burnt out bulbs replaced at the end of 6^{th} week

$= N_0P_6 + N_1P_5 + N_2P_4 + N_3P_3 + N_4P_2 + N_5P_1$

$= 0 + 100 \times 0.20 + 160 \times 0.15 + 281 \times 0.25 + 377 \times 0.15 + 350 \times 0.10$ = 230

N_7 = Number of burnt out bulbs replaced at the end of 7th week

$= N_0P_7 + N_1P_6 + N_2P_5 + N_3P_4 + N_4P_3 + N_5P_2 + N_6P_1$

$= 0 + 0 + 160 \times 0.20 + 281 \times 0.30 + 377 \times 0.25 + 350 \times 0.15 + 350 \times 0.10 + 230 \times 0.10$

$= 230$

And so on........

From the above calculations we see that N_i the expected number of bulbs failing each week increases up - to fourth week and then decreases up to 6th week and then increases again. In this way N_i oscillates till the system settles down to steady state.

The average life of bulb

$= \sum X_i P_i$

$= 1P_1 + 2P_2 + 3P_3 + 4P_4 + 5P_5 + \ldots\ldots\ldots\ldots\ldots\ldots$

$= 0.10 + 2 \times 0.15 + 3 \times 0.25 + 4 \times 0.30 + 5 \times 0.20 = 3.35$

∴ Average number of replacement per week = N / (Mean Age) = 1000/3.35 = 299

∴ Average cost of weekly individual replacement policy = Rs 299

(Since the cost of individual replacement of a bulb cost Rs 1)

Now we consider the case of group replacement

End of week	Total cost of group Replacement in Rs	Average cost per week in Rs
1	$1000 \times 0.25 + 100 \times 1 = 350$	350
2	$1000 \times 0.25 + (100 + 160) \times 1 = 510$	255
3	$1000 \times 0.25 + (100 + 160 + 281) \times 1 = 791$	263.66

Thus, the minimum cost per week is Rs 255.00 if the bulbs are replaced as a group after every two weeks and this cost is also less than the average cost of weekly individual replacement policy.

Hence for minimum monthly cost all the bulbs should be replaced after every two weeks.